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TRANSIENT TEMPERATURES IN HEAT EXCHANGERS FOR SUPERSONIC BLOWDOWN TUNNELS

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SUMMARY

In order to investigate transient temperatures in heat exchangers for supersonic blowdown tunnels, an analysis of the heat transfer between an insulated tube and the fluid moving through the tube was made for three inlet fluid conditions: one in which the fluid temperature was constant, another in which the temperature decreased exponentially, and a third in which the temperature of the fluid decreased linearly with time. For temperatures low enough to permit the neglect of radiation effects, the results of the analytical work give mathematical expressions whereby the tube and fluid temperatures may be computed as functions of axial tube distance and time.

Temperatures were measured for air at a constant inlet temperature and at atmospheric pressure flowing through a bundle of tubes preheated to a constant axial temperature of 202° F. Excellent agreement between the computed and measured temperatures justifies the simplifying assumptions and substantiates the analysis.

INTRODUCTION

One requirement for any intermittent or blowdown tunnel used in the study of jet-engine performance, in the study of heat transfer, or in tests of the structural integrity of airplane structures at supersonic speeds is the establishment of air flow at desired values of stagnation temperature. The large amounts of heat required for this purpose are usually supplied by heat exchangers in the form of bundles of tubes or sheets which are preheated to approximately the desired stagnation temperature of the air. Although sufficient information is available for the aerodynamic design of a supersonic blowdown tunnel such as that described in reference 1, little information is available on the design of a heat exchanger. The present investigation is concerned with the transient heat transfer through a heat exchanger of the heat-accumulator type during operation of a supersonic blowdown tunnel. The effects of radiation are neglected in this study and the application of the results is thus limited to heat exchangers in which the temperatures are not high enough to cause appreciable heat losses by radiation.

Heat exchangers of the heat-accumulator type provide a method of supplying very large amounts of heat for short periods of time while storing energy at a low rate between tests. A wide variety of materials have been considered, from artificially heated rocks at Peenemünde to bundles of steel tubes at the preflight jet of the Langley Pilotless Aircraft Research Station at Wallops Island, Va. In the analysis in this paper, the heat is considered to be stored in bundles of tubes initially preheated to a constant temperature. Several air conditions are examined by mathematical analysis. In the first condition, the air at a constant temperature enters a tube, as reported in reference 2, or a porous medium, as discussed in reference 3. In the second condition, the air is considered to be stored in a tank under pressure. During the test the air from the storage tank undergoes an isentropic expansion before entering the heat-exchanger tubes. The third condition represents a simplification of the preceding condition for an expanding gas in that a linear decrease in the temperature of the air entering the heat exchanger is considered.

The basic laws of heat convection were used to derive an integro-differential equation which expresses the variation of the tube and air temperature difference as a function of time and axial distance along the tube. Upon insertion of the appropriate boundary conditions for each case considered, expressions for the variation of the temperature difference were obtained. Integration of these expressions gave the variation of tube and fluid temperatures with respect to time and axial tube distance.

In order to check the validity of the analytical method and the assumptions involved, an experimental investigation was made for air at a constant inlet temperature and at atmospheric pressure flowing through a tube. Two series of tests were made, one in which a single instrumented tube was placed in the center of a bundle of tubes and one in which the instrumented tube was packed in insulation. The initial temperatures of the tubes were 202° F and 166° F, respectively. The first series of tests represents the situation in a heat exchanger, whereas the second series represents the heating of a fluid passing through a tube where the surrounding medium is much less dense than the tube.

SYMBOLS

A	internal surface area of tube per unit length, sq ft/ft
c_f	specific heat of fluid at constant pressure, Btu/(lb)(°F)

c_p specific heat of tube at constant pressure, Btu/(lb)($^{\circ}$F)

d_{ext} external diameter of tube, ft

d_{int} internal diameter of tube, ft

$$g(t) = \left(\frac{k}{m_0}\right)^{\gamma-1}\left\{\left(\frac{m_0}{k} - t\right)^{\gamma-1} + \frac{K_p}{\gamma}\left(\frac{m_0}{k} - t\right)\left[1 + K_p \frac{\frac{m_0}{k} - t}{\gamma + 1} + \ldots + \frac{K_p{}^n\left(\frac{m_0}{k} - t\right)^n}{(\gamma + 1)(\gamma + 2) \ldots (\gamma + n)} + \ldots\right]\right\}$$

h heat-transfer coefficient, Btu/(sq ft)(sec)($^{\circ}$F)

I_0 Bessel function of zero order

I_n Bessel function of order n

K_f ratio of heat transfer to heat capacity of fluid, 1/sec

K_p ratio of heat transfer to heat capacity of tube, 1/sec

k rate of flow through heat exchanger, lb/sec

l length of tube, ft

m mass of fluid in tank, slugs

m_0 initial mass of fluid in tank, slugs

n integer

q heat flux, Btu/sec

T temperature, $^{\circ}$F

T_f temperature of fluid, $^{\circ}$F

T_{f_0} temperature of fluid at tube entrance, $^{\circ}$F

T_{f_i} temperature of external fluid before flow, $^{\circ}$F

T_{f_t} temperature of fluid at time t, °F

T_p temperature of tube, °F

T_{p_0} initial temperature of tube, °F

T_{p_t} temperature of tube at time t, °F

t time, sec

V fluid velocity, ft/sec

w_f weight of fluid per unit length, lb/ft

w_p weight of tube per unit length, lb/ft

x axial distance along tube, ft (unless otherwise specified)

$$\alpha = 1 + \frac{\sum_{n=0}^{\infty} \left(2K_f \frac{x}{V}\right)^n \left[\sqrt{4K_pK_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right]^{-n} I_n\left[\sqrt{4K_pK_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right]}{e^{K_p\left(t - \frac{x}{V}\right)+K_f \frac{x}{V}}}$$

$$\beta = \frac{1}{e^{K_p\left(t - \frac{x}{V}\right)+K_f \frac{x}{V}}}\left\{1 + K_fK_p \frac{x}{V} + \frac{\left(K_fK_p \frac{x}{V}\right)^2}{2!}\left(- \frac{1}{K_p} + t - \frac{x}{V}\right) + \ldots + \right.$$

$$\left. \frac{\left(K_fK_p \frac{x}{V}\right)^n}{n!}\left[\frac{(-1)^{n-1}}{(n - 1)!K_p^{n-1}} + \frac{(-1)^{n-2}\left(t - \frac{x}{V}\right)}{(n - 2)!1!K_p^{n-2}} + \ldots + \frac{\left(t - \frac{x}{V}\right)^{n-1}}{(n - 1)!}\right] + \ldots\right\}$$

γ ratio of specific heat of fluid at constant pressure to specific heat at constant volume

θ temperature difference between tube and fluid, °F

θ_0 difference between initial temperature of tube and initial temperature of external fluid, °F

θ_i difference between initial temperature of tube and temperature of entering fluid, $^{\circ}$F

ρ density of fluid in tank, slugs/cu ft

ρ_0 initial density of fluid in tank, slugs/cu ft

$$\phi = \frac{I_0\left[\sqrt{4K_p K_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right]}{e^{K_p\left(t - \frac{x}{V}\right) + K_f \frac{x}{V}}}$$

HEAT TRANSFER BETWEEN A PREHEATED TUBE AND THE FLUID

This investigation is concerned with the analysis of heat exchangers of the heat-accumulator type in which the heat source or sink consists of a bundle of tubes through which the working fluid flows. Except for the outer row of tubes where a small amount of heat may be transferred to the surrounding shell, the fluid provides the only means of heat transfer during operation. No heat is transferred across the outer surface of the individual tubes because all are at the same local temperature; that is, each tube is effectively perfectly insulated.

In order to reduce the number of independent variables which must be considered in the analysis, the assumption was made that no radial temperature gradients exist in the tubes and that the temperature difference between the tubes and the fluid occurs across a thin boundary layer. Inasmuch as the conductivity of the tubes is large compared with that of the boundary layer, this assumption appears to be justified. For long tubes, turbulent flow may be considered to exist throughout the tube. If the tubes have blunt ends with sharp internal lips, the flow becomes turbulent 3.5 diameters downstream of the entrance (ref. 4). A further assumption of the analysis is that the transfer of heat axially along the tube can be neglected. The validity of this assumption depends upon the temperature gradient along the tubes and upon the ratio of the tube-wall cross-sectional area to the internal surface area. Effectively, the assumption states that the heat transfer is polarized so that heat is transferred only to the fluid.

For an initial tube temperature greater than the fluid temperature, Newton's law for the convection of heat may be written as follows:

$$dq = hA\theta \ dx \tag{1}$$

where q is the heat flux; h, the heat-transfer coefficient between the tubes and the fluid; A, the internal surface area of a tube; and θ, the temperature difference between the tubes and the fluid. Two additional expressions which represent the conservation of energy are obtained and may be expressed as follows:

$$dq = -w_p c_p \frac{\partial T_p}{\partial x} dx \tag{2}$$

$$dq = w_f c_f \frac{dT_f}{dt} dx \tag{3}$$

In these expressions w_p and w_f represent the weights of the tube and fluid per unit length, c_p and c_f represent the specific heats of the tube and fluid, and T_p and T_f represent the temperatures of the tube and fluid, respectively. A graphical representation of the temperatures and temperature derivatives is presented in figure 1.

In order to obtain the tube and fluid temperatures, an expression must first be obtained for the dependent variable θ in terms of the independent variables x and t. Inasmuch as θ represents the heat flow potential and the fluid and tube temperatures can be expressed as functions of the temperature difference, this selection of the dependent variable was a natural one.

The total differential for the temperature difference θ is

$$\frac{d\theta}{dt} = \frac{\partial T_p}{\partial x}\frac{dx}{dt} + \frac{\partial T_p}{\partial t} - \frac{dT_f}{dt} \tag{4}$$

By combining equation (1) with equations (2) and (3), the following expressions for the temperature derivatives are obtained:

$$\frac{\partial T_p}{\partial t} = -\frac{hA}{w_p c_p}\theta \tag{5}$$

and

$$\frac{dT_f}{dt} = \frac{hA}{w_f c_f}\theta \tag{6}$$

In order to shorten the mathematical expressions used subsequently in this paper, the terms $hA/w_f c_f$ and $hA/w_p c_p$ are replaced by the coefficients K_f and K_p, respectively. Physically these terms represent the ratios of heat transfer between the tube and the fluid to the heat capacities of the fluid and the tube, respectively. In order to linearize the differential equations, K_f and K_p are assumed to be constants throughout the length of the tube. The partial derivative of the tube temperature may be obtained by differentiating equation (5) with respect to x and integrating with respect to time. Thus,

$$\frac{\partial T_p}{\partial x} = -K_p \int_0^t \frac{\partial \theta}{\partial x}\, dt \tag{7}$$

The values of the derivatives of the tube and fluid temperatures, equations (5), (6), and (7), are substituted into equation (4) in order to obtain an expression for θ in terms of x and t:

$$\frac{d\theta}{dt} = -K_p \frac{dx}{dt} \int_0^t \frac{\partial \theta}{\partial x}\, dt - K_p\theta - K_f\theta$$

The fluid velocity V can be substituted for dx/dt. Expanding $d\theta/dt$ gives the expression for the dependent variable θ in terms of the independent variables x and t as

$$V \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial t} = -K_p V \int_0^t \frac{\partial \theta}{\partial x}\, dt - \left(K_p + K_f\right)\theta \tag{8}$$

A further condition necessary to linearize the differential equation is that the fluid velocity remain constant throughout the tube length. Rigidly then, the analysis is restricted to an incompressible fluid. The solution to this integrodifferential equation is the key to the analysis. Once θ has been determined, the tube and fluid temperatures may be obtained from equations (5) and (6) by integration.

CALCULATION OF TUBE AND FLUID TEMPERATURES

Fluid at a Constant Inlet Temperature Passing Through a Tube Initially at a Constant Temperature

Boundary conditions.- The heat-transfer condition considered in this section is one in which a fluid, either gas or liquid, at a constant inlet temperature flows through a tube preheated to an initial temperature which is constant along the length of the tube. A practical example of this condition is furnished in reference 1 in which is discussed a supersonic blowdown tunnel powered by an ejector which takes in air at room temperature. The boundary conditions used in the solution of equation (8) are determined from physical considerations of heat and fluid flow.

The tube is at a constant axial temperature T_{p_0} before the fluid outside the tube, which is at a constant temperature T_{f_i}, starts to flow. Before the test starts, that part of the fluid within the tube attains the tube temperature. Thus, the first boundary condition may be written

$$\theta = 0 \qquad \left(t < 0;\ 0 \leqq x \leqq l\right) \tag{9}$$

The second boundary condition is obtained by considering the temperature relationship at the tube inlet during the test. If no heat is transferred axially along the tube, as is assumed in the preceding section, a differential equation for the temperature difference at the inlet can be obtained. This equation is

$$\frac{d\theta}{dt} = -K_p\theta \tag{10}$$

When $t = 0$, the temperature difference θ_i at the tube inlet is the difference between T_{p_0} and T_{f_i}. When this condition is put into the solution of equation (10), the second boundary condition is determined as

$$\theta = \theta_i e^{-K_p t} \qquad \left(t \geqq 0;\ x = 0\right) \tag{11}$$

Evaluation of θ.- The integrodifferential equation for θ as a function of x and t and the boundary conditions for a fluid with a constant inlet temperature determined in the previous sections are now used to evaluate θ. Equations (8), (9), and (11) are restated for convenience:

$$V \frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial t} = -K_p V \int_0^t \frac{\partial\theta}{\partial x}\, dt - \left(K_p + K_f\right)\theta$$

$$\theta = 0 \qquad \left(t < 0;\ 0 \leqq x \leqq l\right)$$

$$\theta = \theta_i e^{-K_p t} \qquad \left(t \geqq 0;\ x = 0\right)$$

This series of equations is amenable to solution by the Laplace transformation. Values of θ are given as follows:

$$\left.\begin{aligned} \theta &= 0 && \left(t \leqq \frac{x}{V}\right) \\ \theta &= \theta_i \phi && \left(t - \frac{x}{V} > 0\right) \end{aligned}\right\} \qquad (12)$$

where

$$\phi = \frac{I_0\left[\sqrt{4K_p K_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right]}{e^{K_p\left(t - \frac{x}{V}\right) + K_f \frac{x}{V}}}$$

Equations (12) can be shown to be a solution of the integrodifferential equation by direct substitution. The first boundary condition, equation (9), is satisfied by the form of solution given by the Laplace transformation. When x is set equal to zero, the second boundary condition is satisfied. Values of the Bessel function are given in references 5 and 6. In order to provide an easy method of determining θ, values of ϕ are presented in figure 2 as functions of $K_p\left(t - \frac{x}{V}\right)$ for various values of $K_f \frac{x}{V}$.

Tube and fluid temperatures.- After θ has been determined, the tube and fluid temperatures can be calculated from equations (5) and (6). Although both tube and fluid temperatures may be found by integration of these equations, the tube temperatures are the easier ones to obtain. The fluid temperatures are most easily determined by subtracting θ from the tube temperatures.

If a fixed position on the tube is considered, the partial derivative of T_p may be changed to the total derivative and equation (5) can be written in the form

$$\int_{T_{p_0}}^{T_p} dT_p = -K_p \int_0^t \theta \, dt$$

$$T_p = T_{p_0} - K_p \theta_i e^{\left(K_p - K_f\right)\frac{x}{V}} \int_0^t e^{-K_p t} I_0\left[\sqrt{4K_pK_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right] dt$$

This expression can be integrated by parts and the value of T_p then becomes

$$T_p = T_{p_0} - \theta_i \alpha \tag{13}$$

where

$$\alpha = 1 + \frac{\sum_{n=0}^{\infty} \left(2K_f \frac{x}{V}\right)^n \left[\sqrt{4K_pK_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right]^{-n} I_n\left[\sqrt{4K_pK_f \frac{x}{V}\left(t - \frac{x}{V}\right)}\right]}{e^{K_p\left(t - \frac{x}{V}\right) + K_f \frac{x}{V}}}$$

As stated previously, the fluid temperature T_f is obtained by subtracting θ (eq. (12)) from T_p (eq. (13)). Thus,

$$T_f = T_{p_0} - \theta_i(\alpha + \phi) \tag{14}$$

Computed values of α are presented in figure 3 as functions of $K_p\left(t - \frac{x}{V}\right)$ for various values of $K_f \frac{x}{V}$. Either equations (13) and (14) and the tables of reference 6 or values of ϕ and α obtained from figures 2 and 3 may be used to compute the temperatures fairly rapidly for any time and any station along the tube.

Gas Expanded Isentropically in Storage Tank and Flowing Through a Preheated Tube

Boundary conditions.- In the general case of supersonic blowdown tunnels, an endeavor is made to keep the flow properties, pressure and temperature, constant in the test section during each test. When this condition is achieved, the gas used in the test is removed at a constant rate k from the tanks where it is stored under pressure. The relationship between the initial mass of gas in the tank m_0 and the mass at any time during the test m may be written

$$\frac{m}{m_0} = 1 - \frac{kt}{m_0}$$

If no heat transfer to the gas in the tank occurs, the gas in the tank undergoes an isentropic expansion. The mass ratio in this case is related to the gas temperature ratio as follows:

$$\frac{m}{m_0} \propto \frac{\rho}{\rho_0} = \left(\frac{T_{f_0}}{T_{f_i}}\right)^{\frac{1}{\gamma-1}}$$

From these equations the stagnation temperature of the gas during the test is obtained as

$$T_{f_0} = T_{f_i}\left(1 - \frac{kt}{m_0}\right)^{\gamma-1} \tag{15}$$

Usually a valve is located between the storage tank and the heat exchanger to control the flow rate and to deliver the gas to the heat exchanger at a constant pressure. If the gas undergoes a throttling process in the valve, the stagnation temperature of the gas entering the tubes of the heat exchanger is the same as that of the gas in the tank. Furthermore, the flow velocity through the heat exchanger is low (in order to reduce pressure losses through the exchanger) and the static temperature is approximately the same as the stagnation temperature.

From this consideration of the gas temperature at the tube inlet, the first boundary condition can be determined. Initially the tube has a constant axial temperature T_{p_0} and the gas in the tank has a temperature T_{f_i}. When the gas starts flowing through the tube, the gas temperature at the inlet T_{f_0} is defined by equation (15). From equations (5) and (15), a differential equation is written for the temperature difference at the tube entrance:

$$\frac{d\theta}{dt} = -K_p\theta + (\gamma - 1)\frac{T_{f_0}k}{m_0}\left(1 - \frac{kt}{m_0}\right)^{\gamma-2} \tag{16}$$

The value of θ is obtained from equation (16) and the conditions at the tube inlet are applied in order to obtain the first boundary condition

$$\theta = \left[\theta_i + T_{f_i}g(0)\right]e^{-K_pt} - T_{f_i}g(t) \qquad \left(t \geqq 0;\ x = 0\right) \tag{17}$$

where

$$g(t) = \left(\frac{k}{m_0}\right)^{\gamma-1}\left\{\left(\frac{m_0}{k} - t\right)^{\gamma-1} + \frac{K_p}{\gamma}\left(\frac{m_0}{k} - t\right)\left[1 + K_p\frac{\frac{m_0}{k} - t}{\gamma + 1} + \frac{K_p^{\,2}\left(\frac{m_0}{k} - t\right)^2}{(\gamma + 1)(\gamma + 2)} + \ldots + \frac{K_p^{\,n}\left(\frac{m_0}{k} - t\right)^n}{(\gamma + 1)(\gamma + 2)\ldots(\gamma + n)} + \ldots\right]\right\}$$

Before the test starts, the gas inside the tube attains the same temperature as that of the tube and the boundary condition is the same as that expressed by equation (9).

Evaluation of θ.- In order to evaluate θ, a solution must be found for equations (8), (9), and (17) which are

$$V \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial t} = -K_p V \int_0^t \frac{\partial \theta}{\partial x} dt - \left(K_p + K_f\right)\theta$$

$$\theta = 0 \qquad \left(t < 0;\ 0 \leqq x \leqq l\right)$$

$$\theta = \left[\theta_i + T_{f_i} g(0)\right] e^{-K_p t} - T_{f_i} g(t) \qquad \left(t \geqq 0;\ x = 0\right)$$

The easiest method of solution is that of the Laplace transformation. The solution for θ is as follows:

$$\left.\begin{aligned}
&\theta = 0 && \left(t - \frac{x}{V} \leqq 0\right) \\
&\theta = \left[\theta_i + T_{f_i} g(0)\right]\phi - e^{-K_f \frac{x}{V}}\left(g\left(t - \frac{x}{V}\right) + e^{-K_p\left(t - \frac{x}{V}\right)}\left\{K_p K_f \frac{x}{V} + \right.\right. \\
&\quad \frac{\left(K_p K_f \frac{x}{V}\right)^2}{2!}\left[g\left(-K_p\right) + \left(t - \frac{x}{V}\right)\right] + \ldots + \frac{\left(K_p K_f \frac{x}{V}\right)^n}{n!}\left[\frac{g\left(-K_p\right)^{n-1}}{(n-1)!} + \right. \\
&\quad \frac{g\left(-K_p\right)^{n-2}}{(n-2)!}\frac{\left(t - \frac{x}{V}\right)}{1!} + \ldots + \frac{g\left(-K_p\right)\left(t - \frac{x}{V}\right)^{n-2}}{1!(n-2)!} + \\
&\quad \left.\left.\left.\frac{\left(t - \frac{x}{V}\right)^{n-1}}{(n-1)!}\right] + \ldots\right\}\right) && \left(t - \frac{x}{V} > 0\right)
\end{aligned}\right\} \qquad (18)$$

The first term of the expression for θ $\left(\text{for } t - \frac{x}{V} > 0\right)$ can be seen to be similar to that for fluid flow at a constant tube inlet temperature, whereas the second term provides a factor which compensates for the variation in inlet temperature. The last part of the expression for θ converges fairly slowly and necessitates laborious computations involving very large numbers. Because θ cannot be simply expressed as functions of $K_p\left(t - \frac{x}{V}\right)$ and $K_f \frac{x}{V}$ and because the computing machines available do not carry a sufficient number of significant numbers, curves defining these functions were omitted.

Tube and fluid temperatures.- The tube and fluid temperatures can be calculated by integration of equations (5) and (6), respectively. Inasmuch as an analytical integration seems impractical because the expressions are already complicated, it is advisable to determine the temperatures by graphical integration. The easiest temperature to determine was found to be the gas temperature. Thus,

$$T_f = T_{f_i} + \frac{hA}{c_f w_f} \int_0^x \theta \, dx \tag{19}$$

The tube temperature is determined by adding θ obtained from equations (18) to the gas temperature obtained from equation (19).

A Gas With a Linear Temperature Decrease Flowing Through a Tube Initially at a Constant Temperature

Boundary conditions.- In the preceding section, the problem of evaluating the temperatures of a tube and gas for the case where compressed gas from a storage tank flowed through a heat exchanger was analyzed. The variation of the stagnation temperature of the gas during the test is expressed by equation (15). Use of these values produced expressions for T_p and T_f which are difficult to evaluate. In order to provide a simpler method of estimating T_p and T_f, the variation of gas temperature with time may be considered to be linear if a reasonable length of time is considered, that is, one-fifth or one-quarter of the running time allowed by the pressure energy of the stored gas; the variation can thus be expressed in the form

$$T_{f_0} \doteq T_{f_i}\left(1 - \frac{kt}{m_0}\right) \tag{20}$$

Initially the tube has a constant axial temperature T_{p_0} and the stored gas a temperature T_{f_i}. From equation (5), an equation is obtained for θ at the inlet of the tube as

$$\frac{d\theta}{dt} = -K_p\theta - \frac{kT_{f_i}}{m_0} \tag{21}$$

At the start of the test the temperature difference is the constant θ_i. When θ is obtained from equation (21), the first boundary condition is determined so that

$$\theta = \left(\theta_0 + \frac{k}{m_0K_p} T_{f_i}\right)e^{-K_pt} - \frac{k}{m_0K_p} T_{f_i} \qquad \left(t \geqq 0;\ x = 0\right) \quad (22)$$

The second boundary condition is determined in a preceding section and is expressed by equation (9).

Evaluation of θ.- A flow condition exists such that a gas at constant mass rate with a linearly decreasing temperature flows through a tube preheated to a constant axial temperature. The problem is restated in mathematical terms (eqs. (8), (9), and (22)) for convenience as follows:

$$V \frac{\partial\theta}{\partial x} + \frac{\partial\theta}{\partial t} = -K_pV \int_0^t \frac{\partial\theta}{\partial x}\, dt - \left(K_p + K_f\right)\theta$$

$$\theta = 0 \qquad \left(t < 0;\ 0 \leqq x \leqq l\right)$$

$$\theta = \left(\theta_0 + \frac{k}{m_0K_p} T_{f_i}\right)e^{-K_pt} - \frac{k}{m_0K_p} T_{f_i}$$

$$\left(t \geqq 0;\ x = 0\right)$$

Solution of these equations for θ was obtained by use of the Laplace transformation. Thus,

$$\left.\begin{aligned} &\theta = 0 && \left(t - \frac{x}{V} \leqq 0\right) \\ &\theta = \left(\theta_0 + \frac{k}{m_0K_p} T_{f_i}\right)\phi - \frac{k}{m_0K_p} T_{f_i}\beta && \left(t - \frac{x}{V} > 0\right) \end{aligned}\right\} \quad (23)$$

where

$$\beta = \frac{1}{e^{K_p\left(t - \frac{x}{V}\right) + K_f \frac{x}{V}}} \left\{ 1 + K_pK_f \frac{x}{V} + \frac{\left(K_pK_f \frac{x}{V}\right)^2}{2!}\left(-\frac{1}{K_p} + t - \frac{x}{V}\right) + \ldots + \right.$$

$$\frac{\left(K_pK_f \frac{x}{V}\right)^n}{n!}\left[\frac{(-1)^{n-1}}{(n-1)!K_p^{\,n-1}} + \frac{(-1)^{n-2}\left(t - \frac{x}{V}\right)}{(n-1)!1!K_p^{\,n-2}} + \ldots + \right.$$

$$\left.\left.\frac{(-1)\left(t - \frac{x}{V}\right)^{n-2}}{1!(n-2)!K_p} + \frac{\left(t - \frac{x}{V}\right)^{n-1}}{(n-1)!}\right] + \ldots \right\}$$

Although the terms in equations (23) are similar in form to those in equations (18), for a gas at a variable inlet temperature the series in the expressions converges fairly rapidly. For this reason computation of θ is relatively easy. Values of ϕ are presented in figure 2 and values of β are presented in figure 4 as functions of $K_p\left(t - \frac{x}{V}\right)$ for various values of $K_f \frac{x}{V}$.

Tube and gas temperatures.- Integrating equations (5) and (6) and using values of θ from equation (23) gives values of tube and fluid temperatures. Graphical integration is mechanically the easier method and the temperatures of the gas are the easier to determine.

TEST EQUIPMENT

In order to verify the applicability of the computed temperatures for the fluid with constant inlet temperature, test equipment was set up to duplicate the conditions occurring in the heat exchanger. Initially the tube is at a constant axial temperature greater than the inlet fluid temperature. Air at room temperature is drawn at constant velocity through the tube by a blower. This situation corresponds to an induction-type tunnel with an open circuit. A schematic diagram of the equipment is presented in figure 5.

The tube mounting used in these tests is illustrated in figure 6. The instrumented tube was a 5/16-inch hard copper tube 96 inches long with an inside diameter of 0.2435 inch. The interior wall of the tube was cleaned before instrumentation to remove oil and scale. In one series of tests a group of 19 tubes was used with the center tube instrumented (fig. 6(a)). For these tests the instrumented tube can be considered to have perfect insulation because the surrounding tubes tend to have the same axial temperature distribution during air flow. The bundle of tubes was placed in a hexagonal balsa jacket and bound with Paraplex impregnated Fiberglas. The diffuser and the entrance bell were also made of Fiberglas. In a second series of tests, a single instrumented tube was placed in a $2\frac{1}{2}$- by 3-inch balsa insulating beam with an air space 7/16 inch in diameter separating the beam from the tube. Five equally spaced wooden washers supported the tube. A mahogany entrance bell and diffuser were used in this installation (fig. 6(b)). In these tests some heat was transferred from the insulating beam to the tube because the tube temperature decreased during the test. In both series of tests, the air inlet velocity was determined from the static pressure measured at the minimum section in the entrance bell.

In order to bring the tube to the desired temperature, the instrumented tube, in the bundle of tubes or in the insulating beam, was suspended in a heating box until the equilibrium test temperature was reached along the tube. The heating box, 96- by 12- by 12-inch inside dimensions, was constructed of wood-framed celotex board. Nichrome wire, spaced along the box to provide a constant axial tube temperature, was used for the heating element.

The air and tube temperatures were measured by thermocouples spaced along the tube. Thermocouple locations for the two installations are given in tables I and II. Iron-constantan thermocouples, 36-gage, were used to measure air temperatures. The thermocouple beads were 0.02 to 0.05 inch in diameter and located along the center line of the tube. Tube temperatures were measured by copper-constantan thermocouples where 30-gage constantan wire was peened into the center of the tube wall. Calibration of the thermocouple material for both types indicated an accuracy to within ±0.25 percent of the temperature range. A motor-driven Lewis selector switch was used to switch thermocouple leads and transmit the individual potential to the recorders. The temperatures were recorded on two single-channel Brown self-balancing potentiometers, one for air temperatures and the other for tube temperatures. The basic accuracy of these instruments was ±0.25 percent of full-scale deflection.

TESTS

In preparation for each test, the tube was placed in the heating box until the test temperature was obtained. The uniformity and magnitude of the temperature were checked by observing the tube thermocouple readings. During this period, the entrance bell was closed. Immediately before each test the duct was opened and the recorders started. Then the blower was started and the air valve set to obtain the desired inlet velocity. Air at room temperature was drawn through the tube and heated. This inlet air remained at essentially a constant temperature. The test procedure was the same for the single insulated tube and the bundle of tubes.

The static pressure in the entrance bell was measured on an electrically driven micromanometer. The thermocouple potential for the tube and fluid were recorded by self-balancing potentiometers. The barometric pressure was used as the value of total pressure during a test.

Data are presented for a bundle of tubes at an inlet velocity of 75 feet per second. The initial tube temperature was 202° F and the initial air temperature was 86° F. The inlet Reynolds number, based on the internal diameter of the tube of 0.2435 inch, was 8,620 for these tests.

For a single tube in insulation, a test was made at an inlet velocity of 103 feet per second and a Reynolds number of 11,780. The initial tube temperature was 166° F and the entering air temperature was 74° F.

ACCURACY

The accuracy of the individual components of the temperature measurement system is given in the section entitled "Test Equipment." Calibrations indicated an error of ±0.25 percent of the temperature range for the thermocouple material and ±0.25 percent of full-scale deflection for the recorders. Thus, mechanically the temperature error totals ±0.50 percent of full-scale deflection. However, a further source of error occurs in printing and reading the records. This error is estimated to be ±0.5° F. Thus, the experimental error involved in the temperature measurements varies from ±2.70 percent for the low temperatures to ±1.00 percent for the high temperatures.

RESULTS AND DISCUSSION

In preceding sections, the heat transfer between a fluid and a tube, initially preheated to a constant axial temperature, was analyzed and the temperature difference θ was determined for a fluid with several types of temperature variations at the tube inlet. In addition to being applicable to bundles of tubes, the results may, when the appropriate heat-transfer coefficients are chosen, be applied to heating elements which form a continuous surface parallel to fluid flow such as plates. Furthermore, large numbers of staggered rods normal to the flow, porous material, and similar heat-accumulator material may also be evaluated. The tube and fluid temperatures were obtained by integration of θ, analytically in the case of a constant fluid inlet temperature and graphically for linearly and isentropically decreasing gas temperatures. In the analysis, heat losses due to radiation were neglected and the results are limited to those cases in which the heat-exchanger temperatures are low enough to justify this condition.

Correlation of computed and experimental temperatures.- In the analysis of the tube and fluid temperatures, one of the major assumptions is that the tube and fluid properties are invariant with time and distance along the tube, that is, independent of temperature variation. It was shown in reference 4 that turbulent flow starts 3.5 diameters from the inlet of blunt-edged tubes. Thus, the use of an average turbulent heat-transfer coefficient (ref. 7) over the entire length of the heat-exchanger tubes is justified in evaluating the coefficients K_p and K_f. From the definitions of these terms, it can be seen that, for all practical purposes, K_p is constant along the tube. However, K_f is a function of the fluid density; for liquids K_f can be considered constant, but for gases a large variation occurs. Two methods of using the fluid properties were tried in computing the coefficients. In one, the properties of air at the tube inlet temperature were used, and in the other, the properties of air were determined for an average temperature between the tube temperature and the air inlet temperature. Of these methods of determining coefficients, use of the average temperature provided closest agreement between experimental and computed temperatures. Inasmuch as the correlation was good over the entire length of the tube and over a fairly long time interval, the assumption that K_f was constant is justified.

In order to check the validity of the analytical method and the assumptions involved, tests were made in which air at a constant entrance temperature was drawn through a tube initially preheated to a constant axial temperature. This condition was tested because of the simplicity of the test equipment required and because, in dealing with a gas, the greatest departure from analytical conditions prevailed.

Multiple-tube bundle.- A comparison of computed and measured tube temperatures is presented in figure 7(a) for an instrumented tube in the center of a bundle of tubes. The computed temperatures presented are based on the properties of air evaluated at the average air temperature. Essentially, the measured temperatures are those of a perfectly insulated tube as required by the analysis. The computed tube temperatures are very close to the experimental values over the entire length of the tube and for the time interval shown. Such agreement seems to justify fully the analysis presented in the preceding sections of this paper.

The values of air temperatures given by equation (14) represent the mean air temperature across the tube. The air thermocouples were located along the center line of the tube and, thus, give the minimum temperature at their respective axial stations. For the purpose of correlating the computed air temperatures, a 1/7-power temperature distribution was taken across the tube because the flow was considered to be turbulent. A relationship between the mean temperature and the minimum air temperature was obtained by integration of the air temperature distribution across the tube. The values of minimum air temperatures at the center of the tube and the mean air temperature are plotted in figure 7(b). The computed and measured temperatures at the center of the tube can be seen to agree except near the tube inlet where a fully developed turbulent temperature distribution does not occur. However, it is beyond the scope of this investigation to determine the temperature distribution across the tube at these stations.

Single insulated tube.- In order to show the magnitude of error involved when heat transfer resulting from imperfect insulation exists, data are presented in figure 8 for a single tube in insulation. The computed tube and air temperatures presented in figures 8(a) and 8(b) are based on the properties of air at the average temperature. Both tube and air temperatures had higher experimental values than corresponding computed values although fair correlation was obtained at the downstream end of the tube. Inasmuch as the heat content of the insulation was not attributed to the tube because of the nonuniformity of the heat flow path, this condition can represent an appreciable error. If a uniform insulation about the tube were used, inclusion of the mass of the insulating material in the coefficient K_p should lower the differences between the computed and the experimental values.

CONCLUDING REMARKS

An analytical method has been presented for the computation of tube and fluid temperatures in a heat exchanger consisting of bundles of tubes preheated to a constant axial temperature. Results have been obtained for three cases of interest in the design of supersonic blowdown tunnels: the cases in which the air inlet temperature is constant, decreases exponentially, and decreases linearly with time of test. Although the conditions of the paper deal with heating, the results can be applied equally well to cooling processes.

Computed and experimental temperatures for air at a constant entrance temperature and at atmospheric pressure flowing through a bundle of tubes preheated to 202° F have shown excellent correlation. This agreement justifies the method of temperature calculation.

For the case of air at a constant entrance temperature and at atmospheric pressure flowing through a single insulated tube preheated to 166° F, the experimental values of temperatures were higher than computed values. However, at the tube exit, the differences were not too great for practical use.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., January 28, 1954.

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TABLE I

LOCATION OF THERMOCOUPLE IN TUBE BUNDLE

Location of thermocouple for measuring temperature of air, x, in.	Location of thermocouple for measuring temperature of tube, x, in.
0	0.125
2	2.125
4	4.125
8	8.125
16	16.125
26	26.125
36	36.125
56	56.125
76	76.125
96	95.875

TABLE II

LOCATION OF THERMOCOUPLE IN SINGLE INSULATED TUBE

Location of thermocouple for measuring temperature of air, x, in.	Location of thermocouple for measuring temperature of tube, x, in.
0	0.125
4	4.125
10	10.125
18	18.125
28	28.125
38	38.125
48	48.125
58	58.125
68	68.125
78	78.125
96	95.875

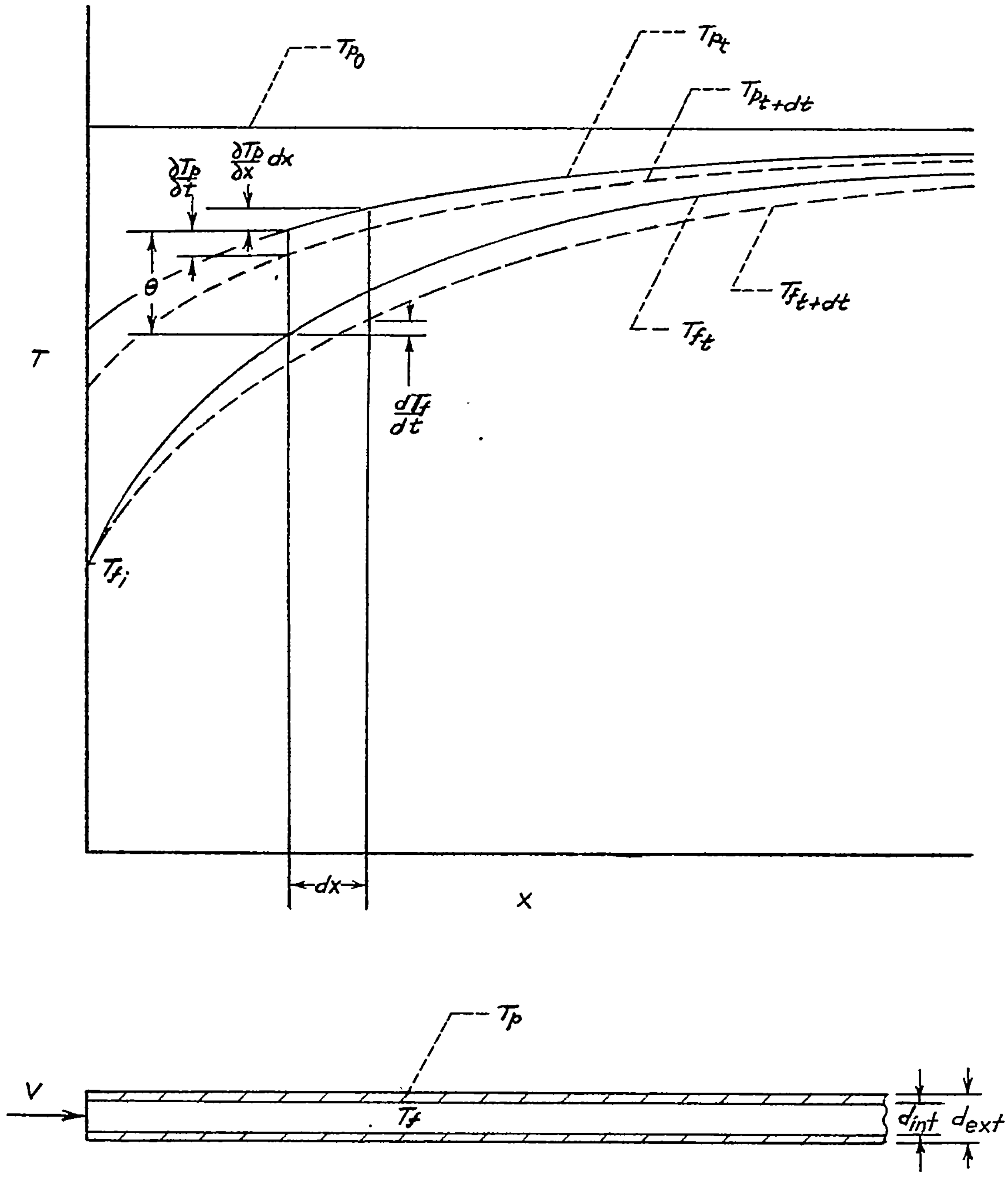


Figure 1.- Graphical illustration of temperature relationships along tube.

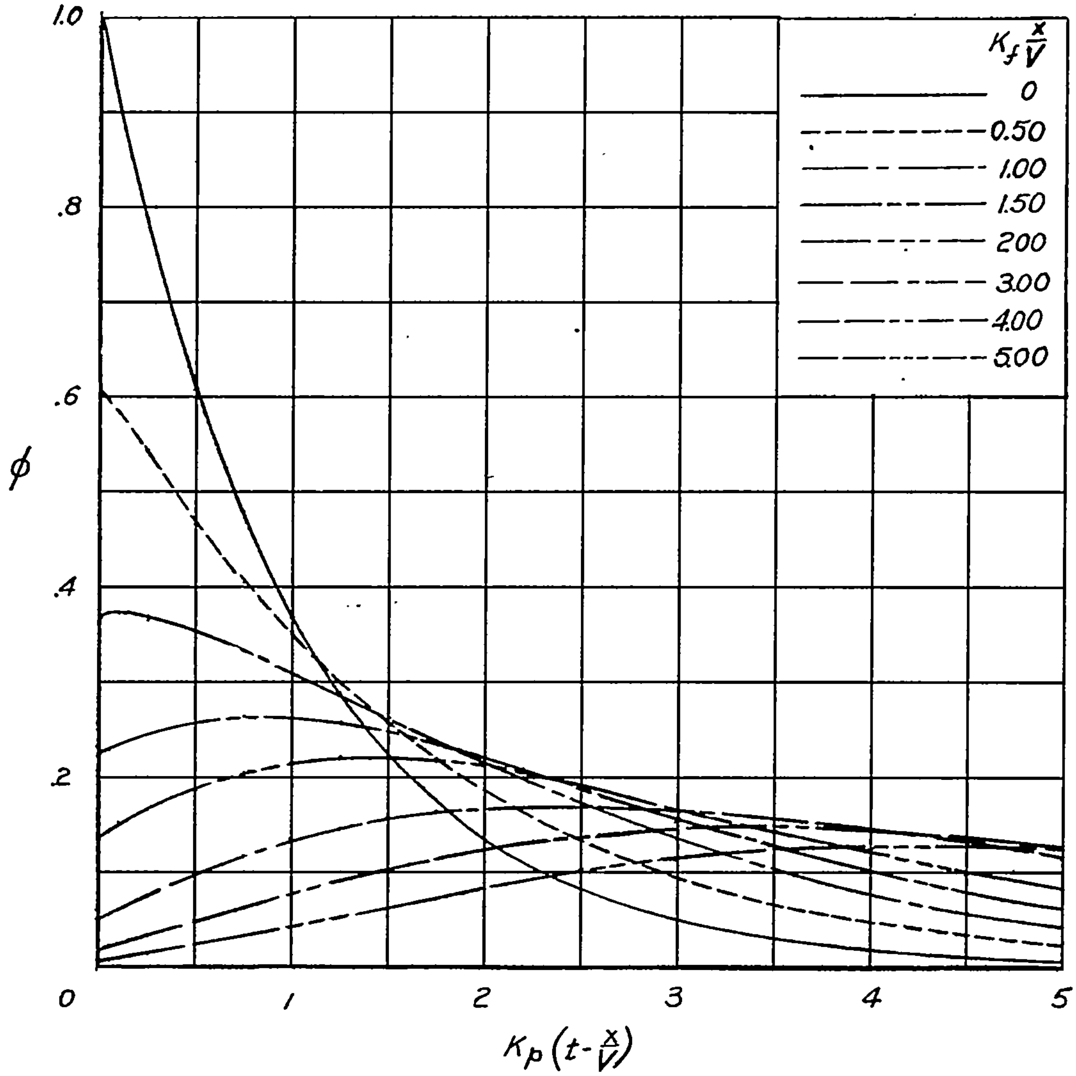


Figure 2.- Variation of ϕ for fluid with a constant inlet temperature.

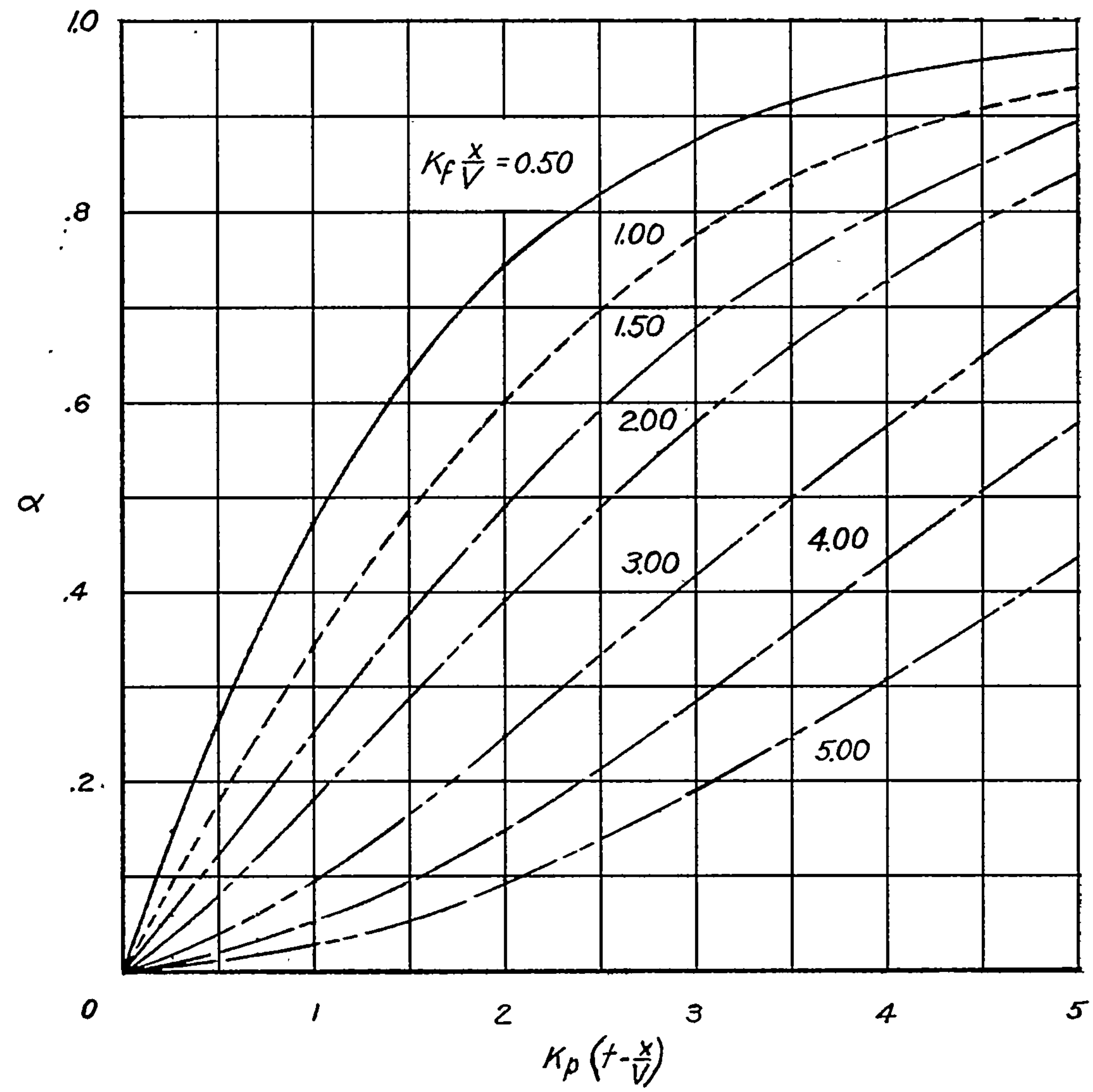


Figure 3.- Variation of α for fluid with a constant inlet temperature.

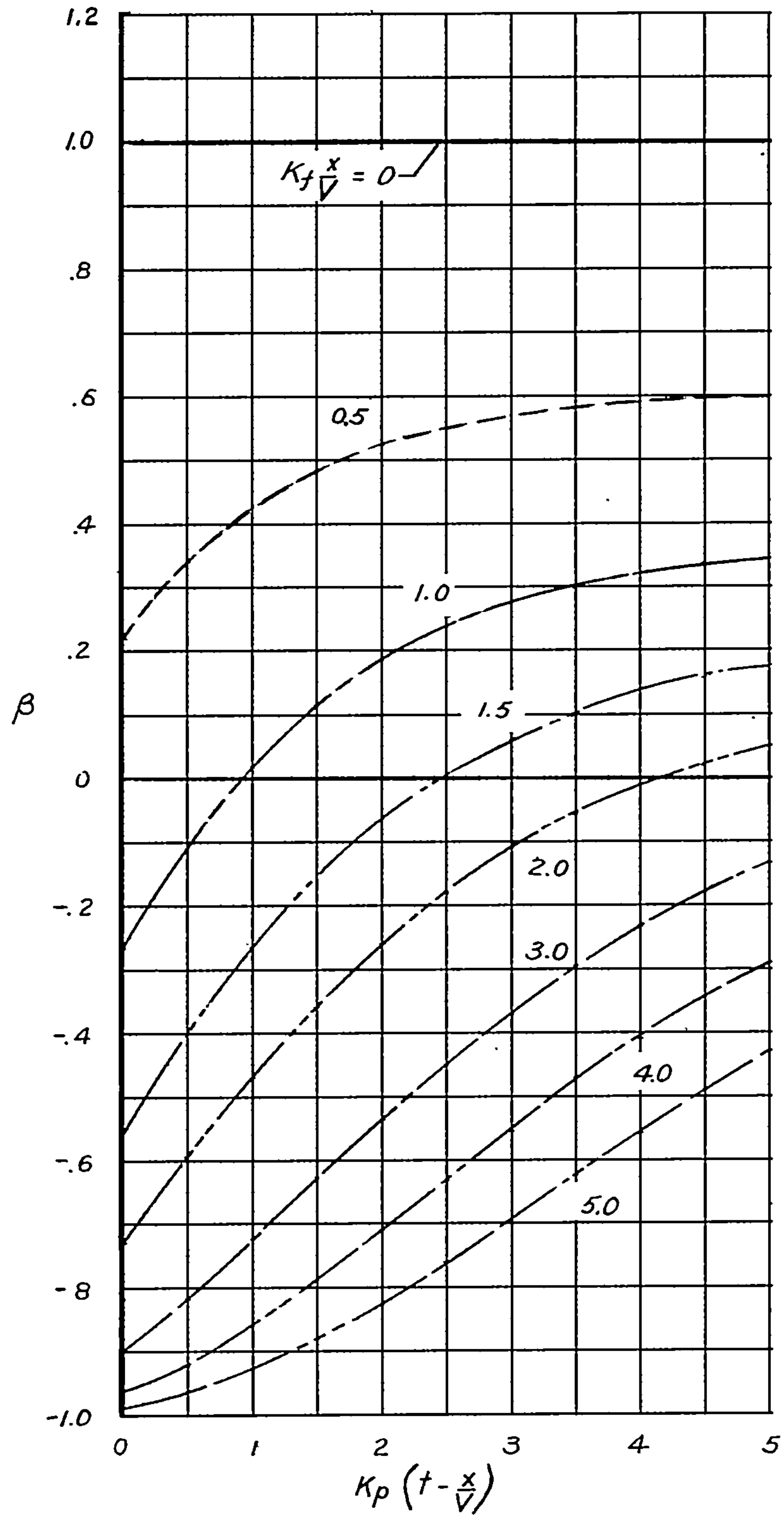


Figure 4.- Variation of β with $K_p\left(t - \frac{x}{V}\right)$ for various values of $K_f\frac{x}{V}$.

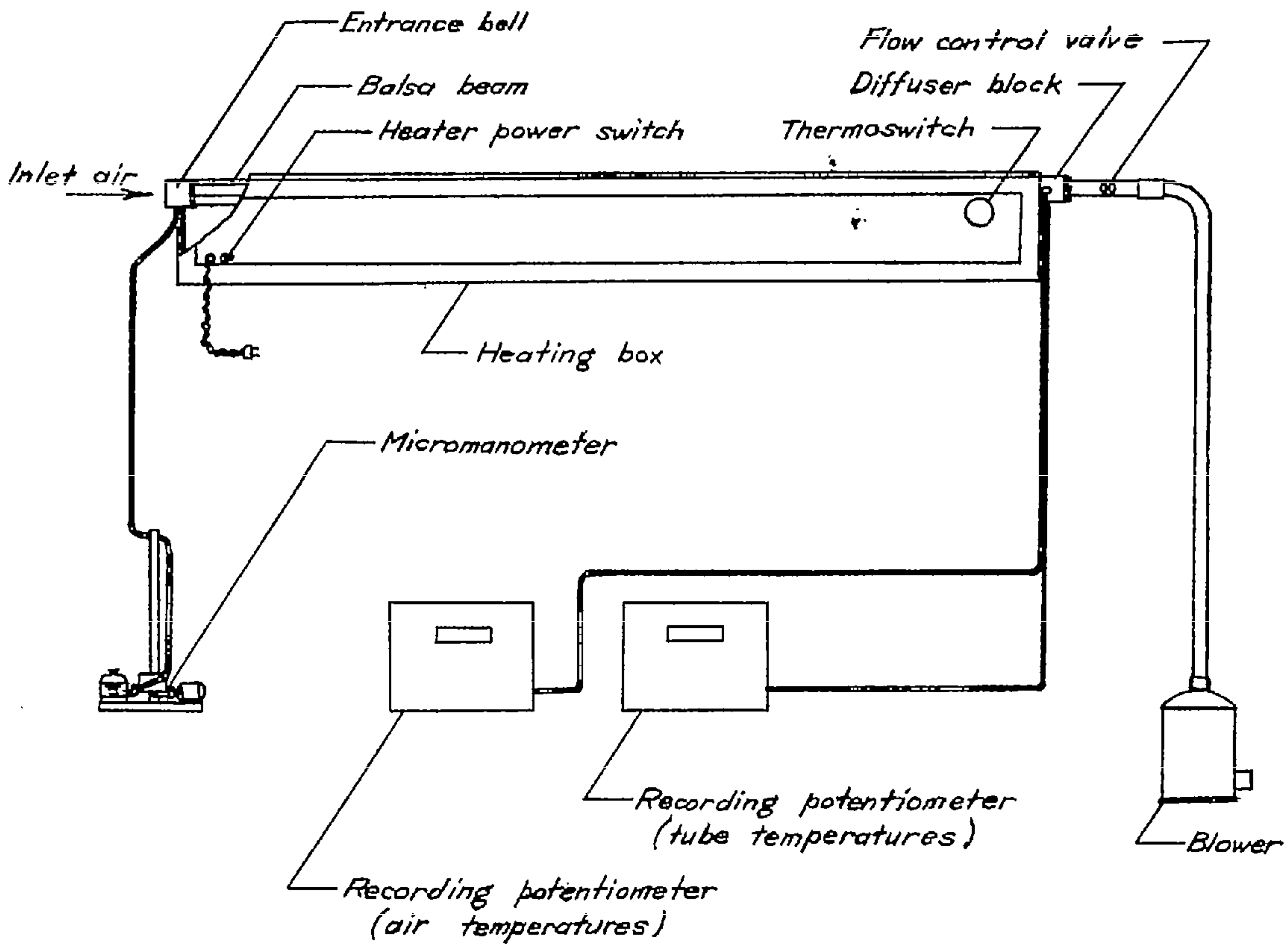
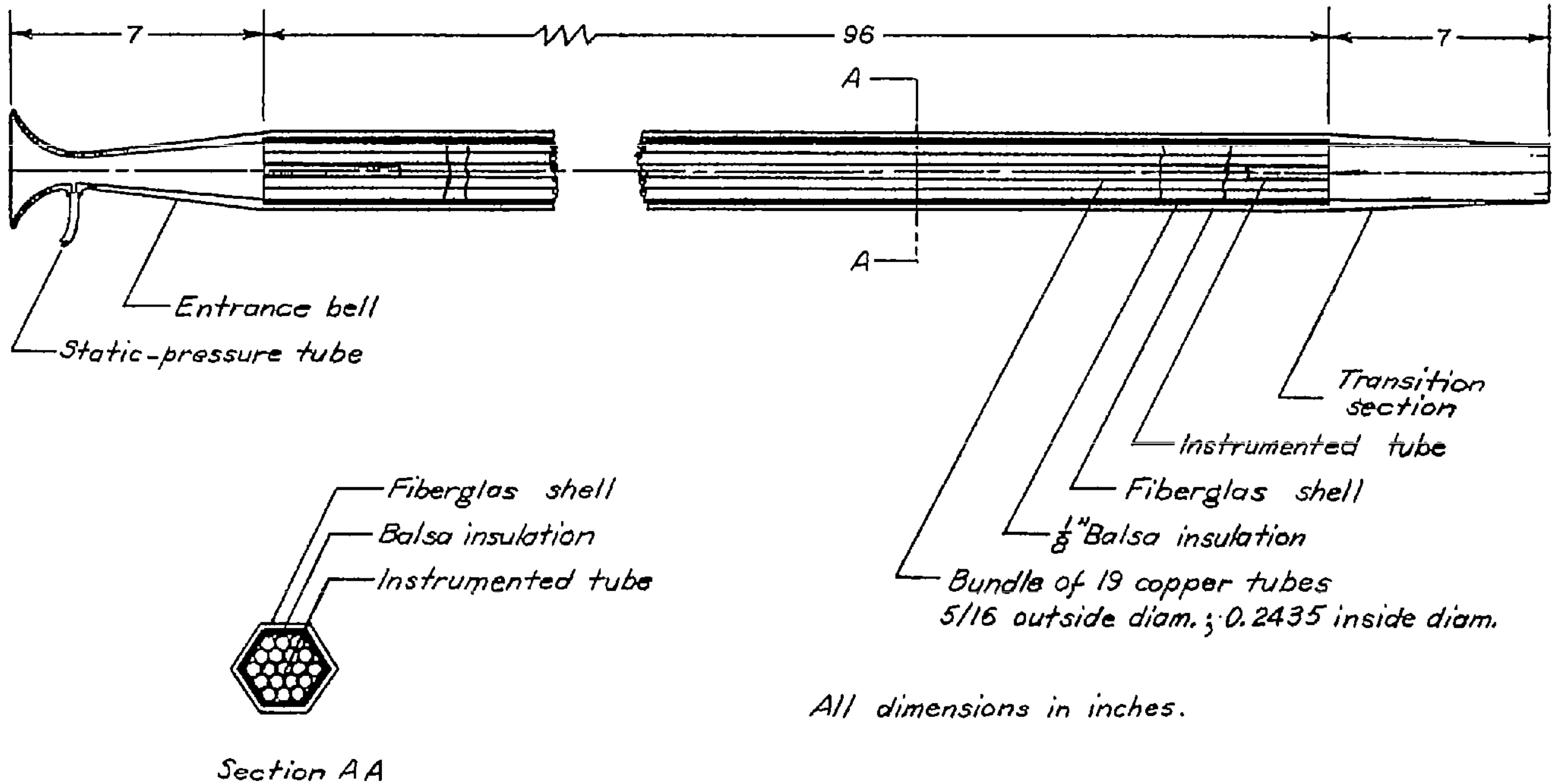
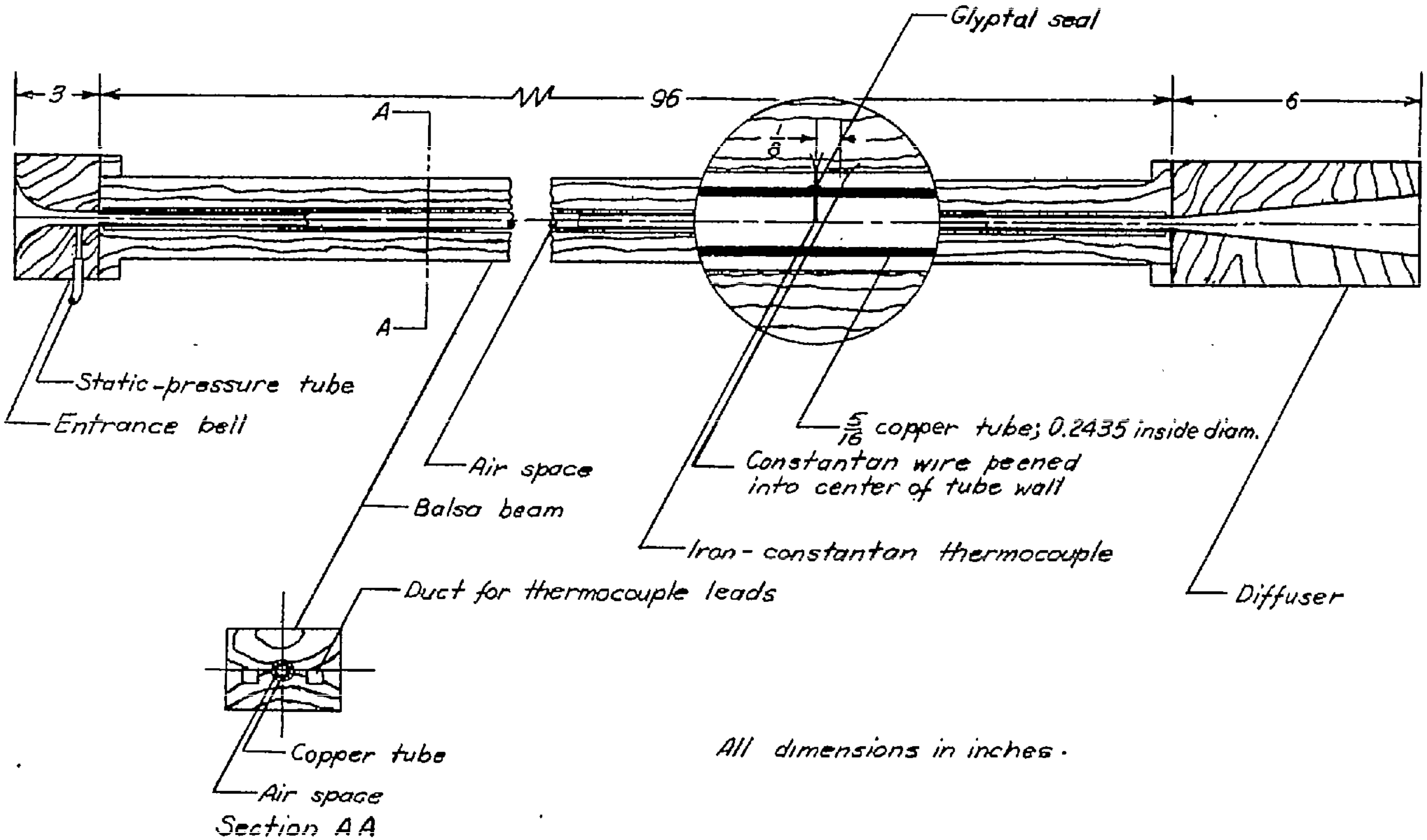


Figure 5.- Diagram of experimental setup for single tube with air flow at constant inlet temperature.



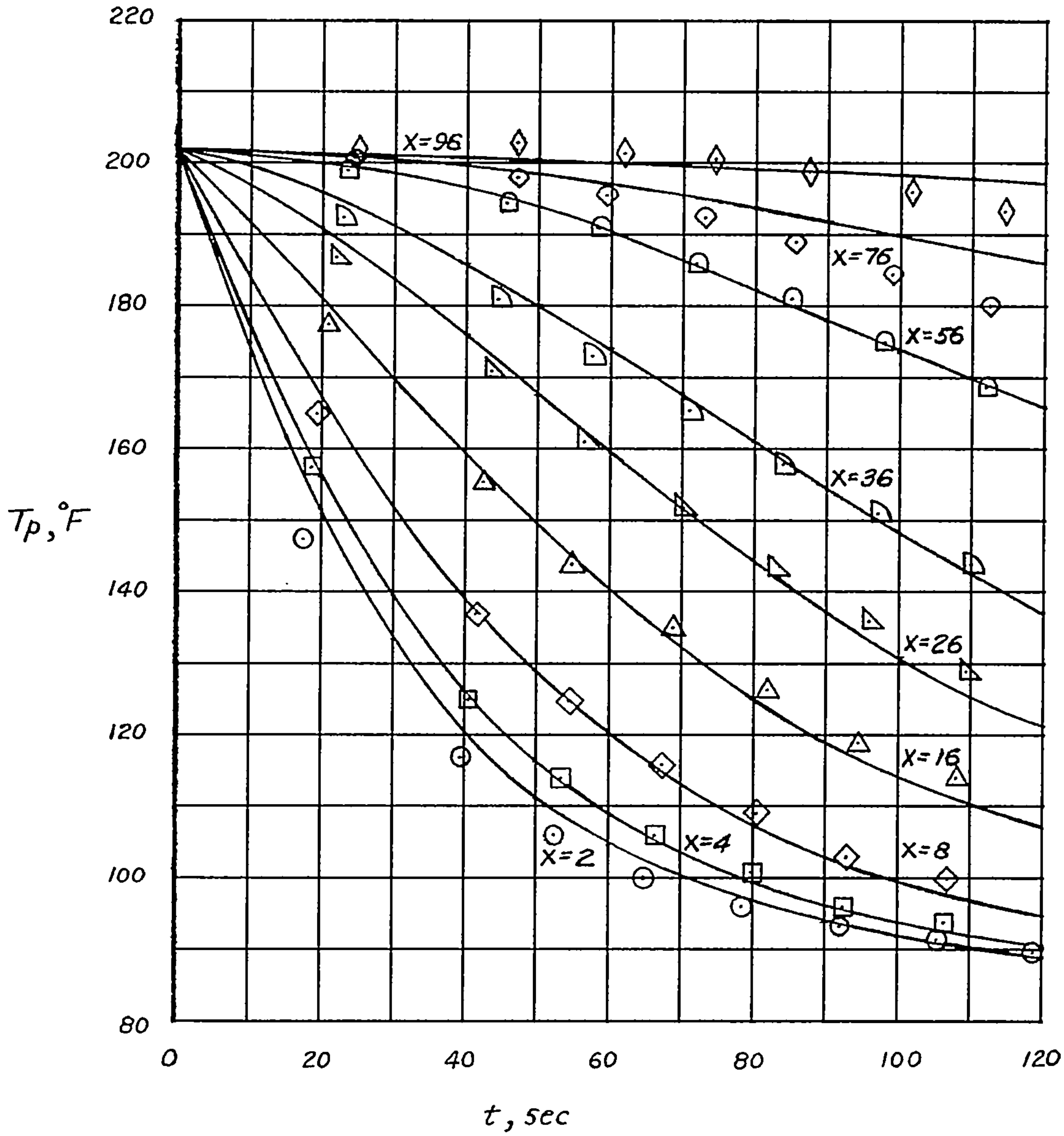
(a) Multiple-tube installation for air flow at constant inlet temperature.

Figure 6.- Illustration of tube mounting.



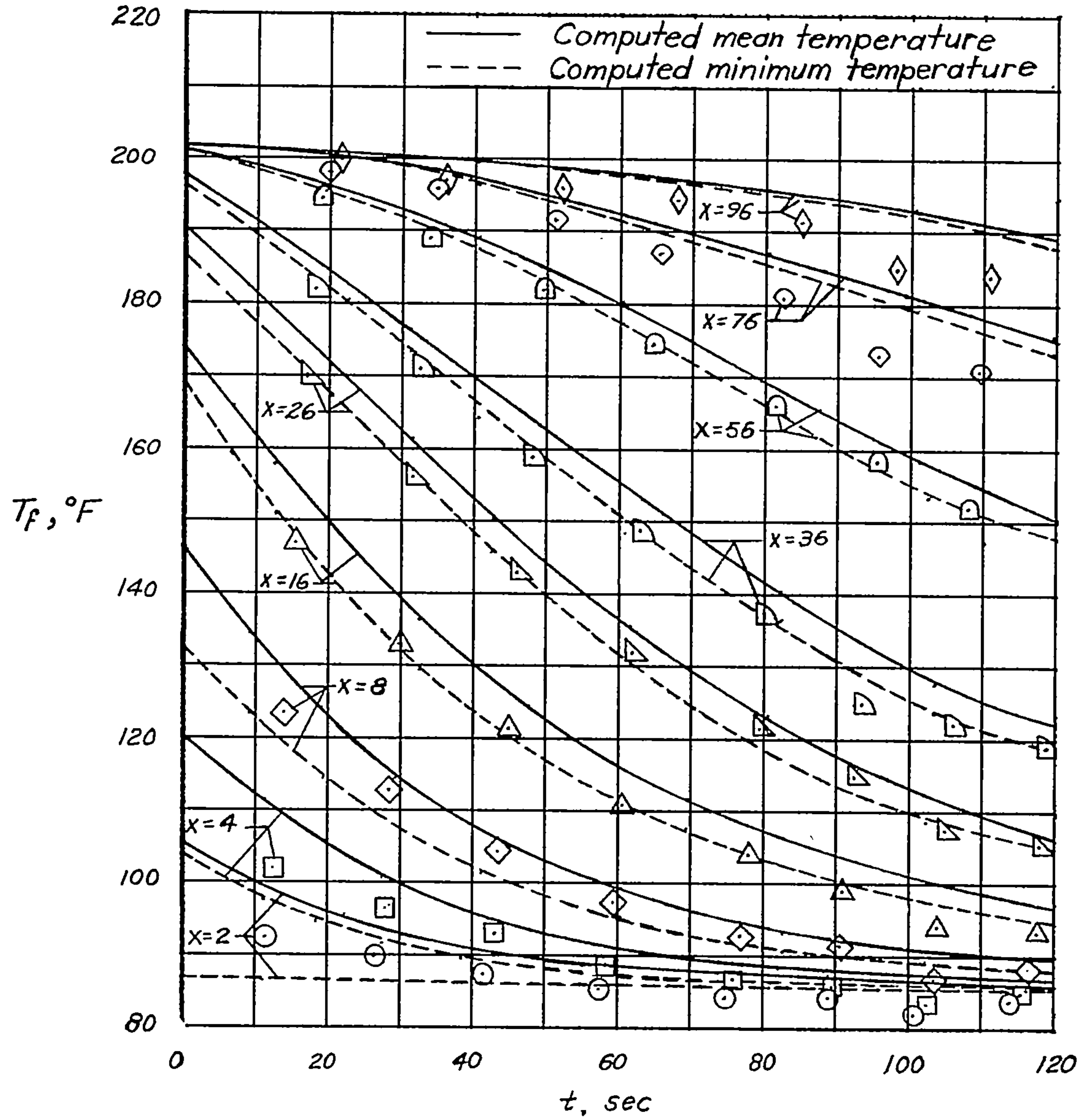
(b) Single-tube installation for air flow at constant inlet temperature.

Figure 6.- Concluded.



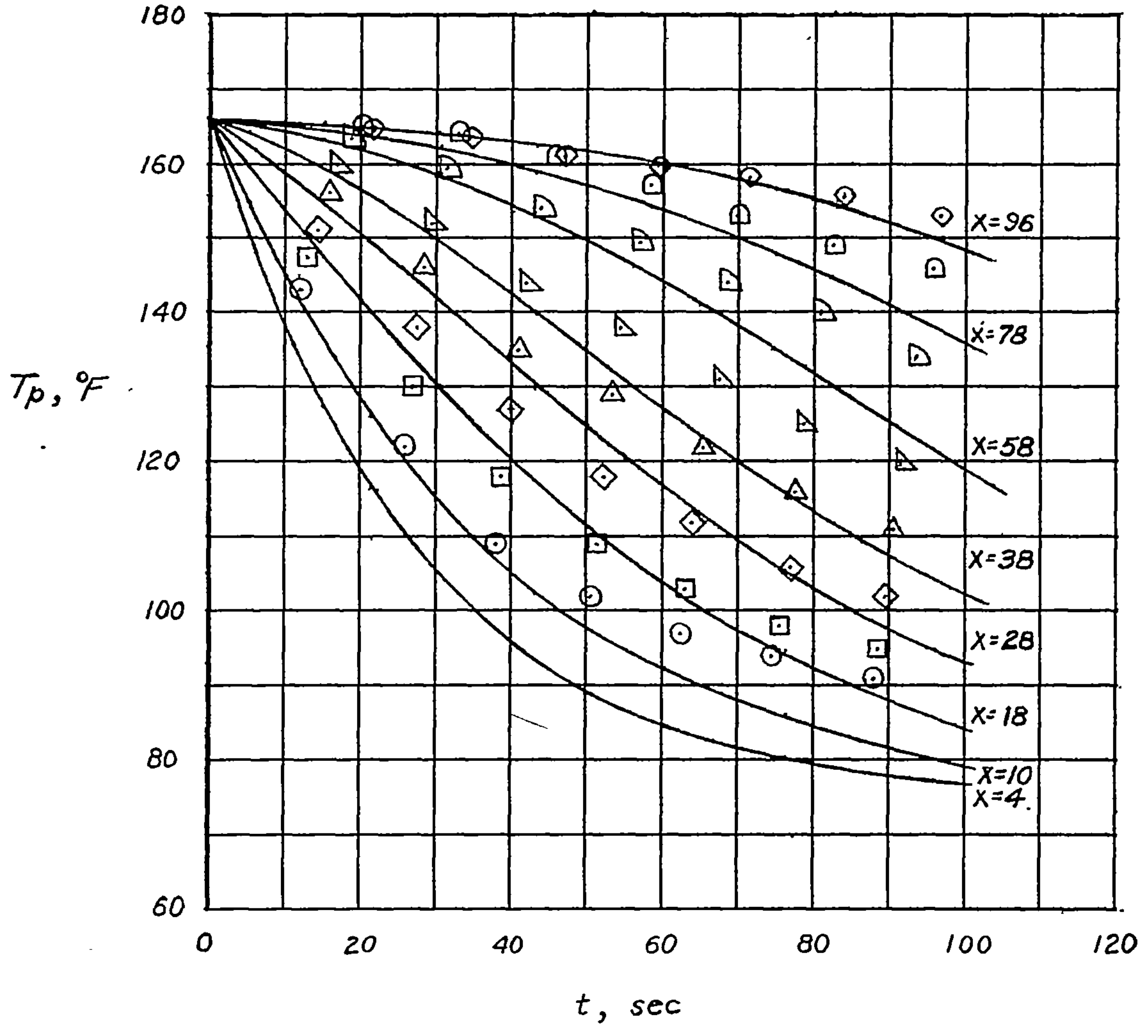
(a) Tube temperatures.

Figure 7.- Comparison of computed and measured temperatures for multiple-tube bundle. (All values of x in inches.)



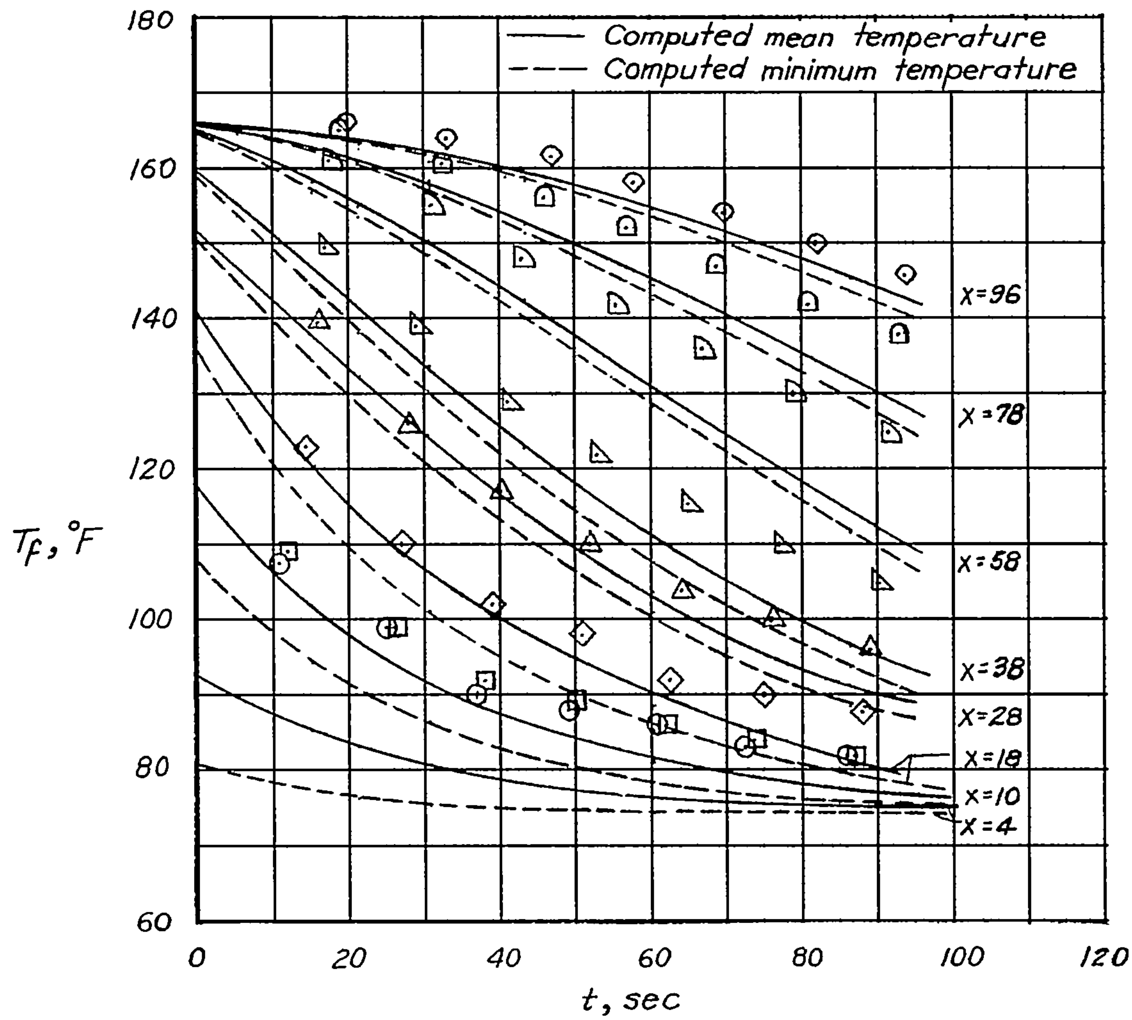
(b) Air temperatures.

Figure 7.- Concluded.



(a) Tube temperatures.

Figure 8.- Comparison of computed and measured temperatures for an insulated tube. (Values of x given in inches.)



(b) Air temperatures.

Figure 8.- Concluded.

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